

# Advanced Microeconomics III

## Spence's Signaling Model

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# Introduction

- **Akerlof's Market for Lemons:** asymmetric information can lead to inefficient market outcomes.
  - Buyers cannot distinguish good from bad sellers.
  - Good sellers might be driven out of the market (adverse selection).
- To overcome adverse selection, good sellers need a way to convey their type.
- **Signaling:** type can be conveyed, but only through indirect observable actions.

- **Examples of signaling:**

- *Warranties*: Firms use them to signal the quality of durable goods.
- *Education*: Workers use it to signal their ability to employers.
- *Advertising*: Companies use to signal product quality.

- **Key questions:**

- How does signaling occur in equilibrium?
- What are the welfare implications of signaling?

# Spence's model

- **Agents:**
  - A single worker and multiple firms (at least 2).
- **Worker Types:**
  - $\theta \in \{\theta_L, \theta_H\}$  with  $\theta_H > \theta_L$ .
  - Only the worker knows  $\theta$ .
  - Firms assign probability  $q$  to type  $\theta_H$ .
- **Production and Payoffs:**
  - If employed by a firm, worker produces output  $\theta$ .
  - Firm's payoff:
    - $\theta - w$  if it employs the worker at wage  $w$ .
    - Zero otherwise.

# Spence's model

- **Timing:**
  - Worker chooses education level  $e \in [0, \infty)$ . This is publicly observed by all firms.
  - Firms make wage offers to the worker.
  - Worker chooses a firm to work for.
- **Worker payoff** when having education  $e$  and employed at wage  $w$ :

$$u(w, e|\theta) = w - c(e|\theta)$$

Where  $c(e|\theta)$  is the cost of education.

- Note that education in this model is unproductive, i.e. it doesn't affect worker's output.

# Spence's model

- **Assumptions on the cost of education:**

- The cost of no education is zero.

$$c(0|\theta) = 0 \quad \text{for all } \theta$$

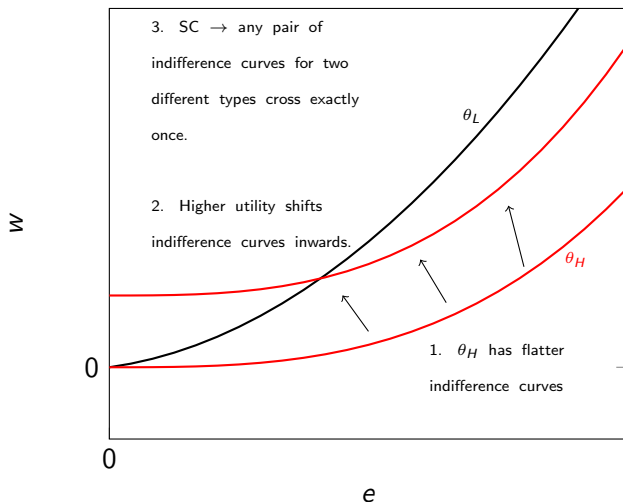
- The cost of education is str. increasing and str. convex for all  $\theta$ .

$$c'(e|\theta) > 0 \quad \text{and} \quad c''(e|\theta) > 0$$

- The high-type worker has a smaller marginal cost of education.

$$c'(e|\theta_H) < c'(e|\theta_L) \quad \forall e > 0 \quad (\text{Single-crossing})$$

# Indifference curves



## Solution concept

- **Solution concept:** Symmetric (Pure-strategy) Perfect Bayesian Equilibrium.
- Consists of:
  - A choice of education level for each worker type:  $e_L, e_H$ .
  - Firms' posterior beliefs about the worker being of type  $H$ :  $\mu(e)$ .
  - Wage offers of the firms:  $w(e)$ .
- Satisfying:
  - Optimal education choice given wage offers.
  - Consistent beliefs whenever possible.
  - Wage offers constitute a Nash equilibrium at each subgame.
  - Firms believe other firms conform to equilibrium wage offer  $w(e)$  both on and off the equilibrium path.
- **Symmetry:** All firms hold the same beliefs after observing education. This is **Not** implied by weak PBE.



- **Wage offers:**

- Competition among firms leads to the following wage offers (why?):

$$w(e) = E_{\mu(e)}[\theta] = \mu(e) \cdot \theta_H + (1 - \mu(e)) \cdot \theta_L$$

- **Education:**

- We distinguish two types of pure-strategy equilibria.
  - **Separating equilibria:**  $e_H \neq e_L$ .
  - **Pooling equilibria:**  $e_H = e_L$ .

## Separating equilibria

- We start characterizing separating equilibria:  $e_H \neq e_L$ .
  - Bayes' rule where possible:

$$\mu(e_L) = 0 \quad \mu(e_H) = 1$$

- By competition:

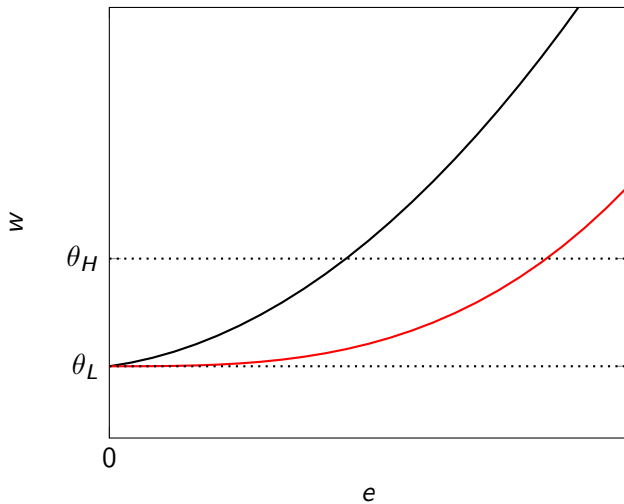
$$w(e_L) = \theta_L \quad w(e_H) = \theta_H$$

### Lemma

In any separating equilibrium,  $e_L = 0$ .

- PBE implies that  $w(e) \in [\theta_L, \theta_H]$ .
- So, if  $e_L > 0$ , the deviation to  $e = 0$  is profitable for type  $\theta_L$ .

## Separating equilibria



## Separating equilibria: incentive compatibility

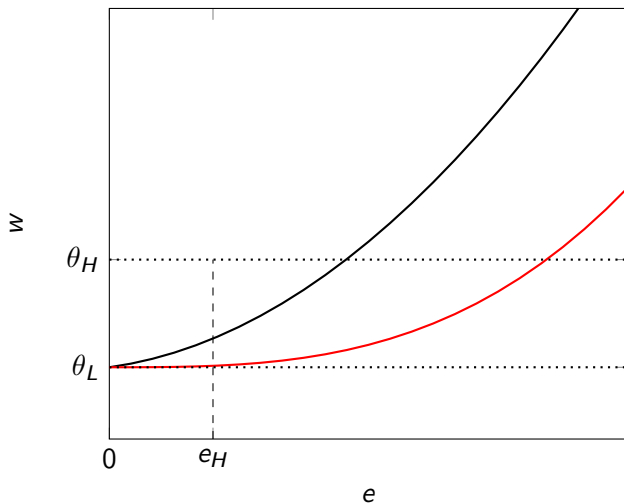
### Lemma

*In a separating equilibrium, type  $H$  chooses  $e_H > 0$  such that*

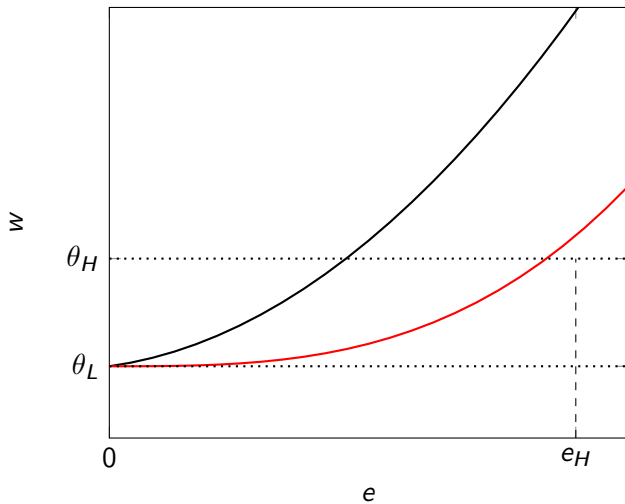
$$\theta_H - c(e_H|\theta_H) \geq \theta_L \geq \theta_H - c(e_H|\theta_L) \quad (\text{IC})$$

- First inequality: type  $H$  prefers his education  $e_H$  rather than zero.
- Second inequality: type  $L$  prefers zero rather than  $e_H$ .

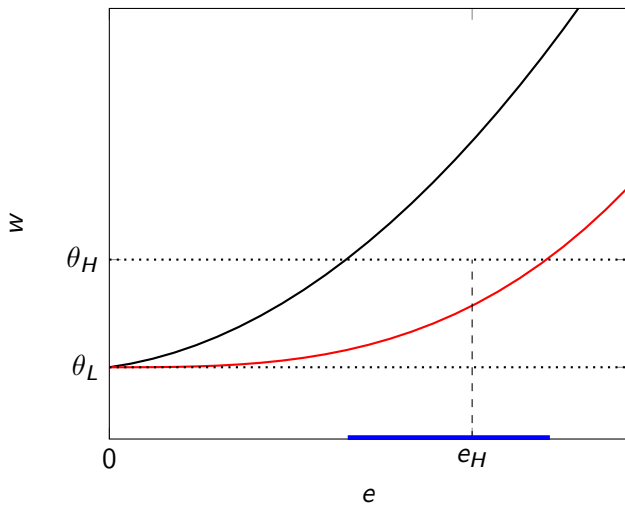
## Separating equilibria: IC



## Separating equilibria: IC



## Separating equilibria: IC



# Separating equilibria

- Previous lemmata describe *necessary* conditions for separating equilibrium.
- These are also *sufficient*: remains to specify out-of-equilibrium beliefs.
  - Deviations are considered to be by a low type:  $\mu(e) = 0$  for all  $e \neq e_H$ .
  - Then, consistent wage is  $\theta_L$  for any  $e \neq e_H$ .
  - Any deviation is unprofitable.



## Equilibrium multiplicity

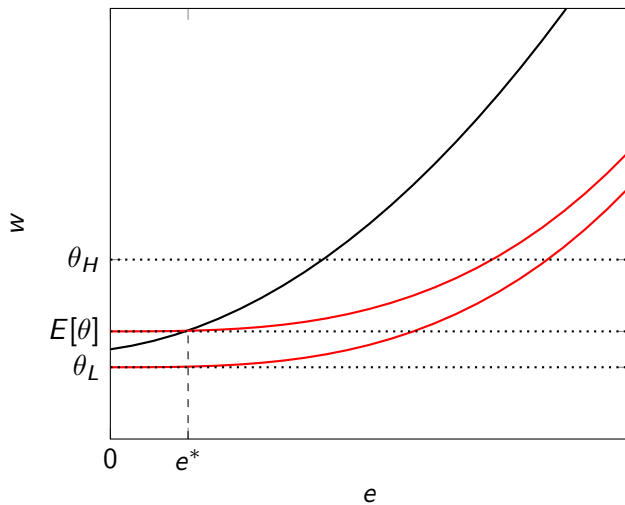
- There are **multiple** separating equilibria.
  - These equilibria can be Pareto ranked.
  - The best separating equilibrium has the lowest education  $e_H$ .

$$c(e_H|\theta_L) = \theta_H - \theta_L$$

# Pooling equilibria

- Pooling equilibrium:  $e_L = e_H = e^*$ .
- Bayes' rule where possible:  $\mu(e^*) = \Pr(\theta = \theta_H) = q$ .
- Competition implies that  $w(e^*) = E[\theta]$ .
- Out-of-equilibrium beliefs:  $\mu(e) = 0$  for  $e \neq e^*$ .
  - Then  $w(e) = \theta_L$  for  $e \neq e^*$ .

# Pooling equilibria



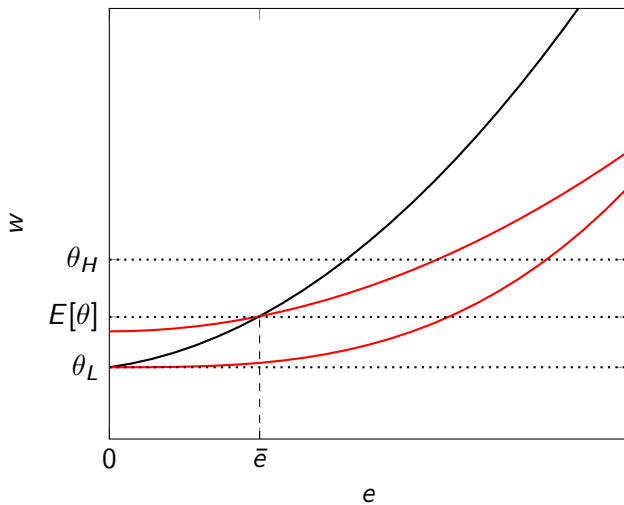
## Multiple pooling equilibria

- There are multiple pooling equilibria.
- The best pooling equilibrium is the one with the lowest level of education ( $e^* = 0$ ).
  
- What about the *worst* one?

$$E[\theta] - c(\bar{e}|\theta_L) = \theta_L$$

$$c(\bar{e}|\theta_L) = E[\theta] - \theta_L$$

## Worst pooling equilibrium



## Comparing pooling and separating equilibria

- The best pooling equilibrium may or may not Pareto dominate the best separating equilibrium.
  - High types not always benefit from the availability of a signaling device. Only if their fraction is small enough.
  
- The best separating equilibrium *never* Pareto dominates the best pooling equilibrium.
  - The low type is always worse-off in a separating equilibrium.

## Reasonable beliefs (equilibrium refinements)

- Which equilibrium is more likely to emerge?
  - Pareto dominance is not a game-theoretical argument.
  
- Forward induction arguments can be used to refine the equilibrium.
  - PBE allows for any beliefs off the equilibrium path.
  - Refinements put conditions on these off equilibrium beliefs.
  - Most refinements in this game uniquely select the least costly separating equilibrium.

## Intuitive criterion

- Cho and Kreps (1987) 'Intuitive criterion':
  - Key question: Who might benefit from the deviation?

### Definition

A deviation  $e'$  is dominated in equilibrium for type  $\theta$  if, for any sequentially rational response by the receivers  $w' = E_{\mu'}[\theta]$  for some beliefs  $\mu'$ , the resulting payoff  $u(e', w', \theta)$  is less than the equilibrium payoff  $u(e(\theta), w(e(\theta)), \theta)$ .

### Definition

A PBE *passes the Intuitive Criterion Test (ICT)* if no type  $\theta$  would be better off deviating to an action  $e' \neq e(\theta)$  should the receivers' beliefs following  $e'$  assign zero probability to types  $\theta'$  for whom the deviation is *dominated in equilibrium*.



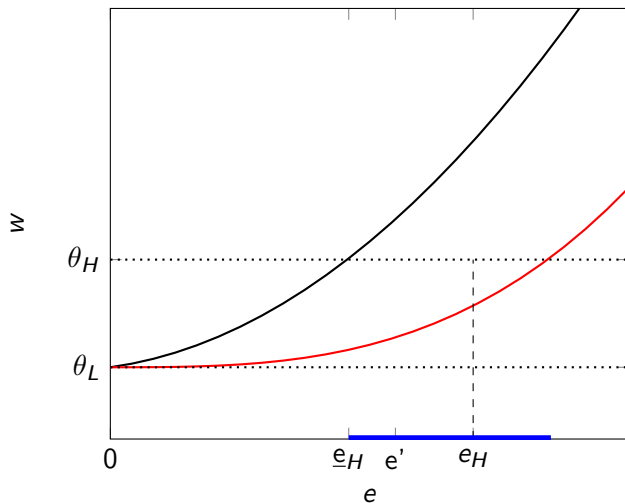
## Intuitive criterion: separating equilibrium

- Let  $\underline{e}_H$  be the minimal high-type education that can be sustained in a separating equilibrium.
- Starting from a separating equilibrium with  $e_H > \underline{e}_H$ , we show that ICT is violated.
  - Consider a deviation to  $e' \in (\underline{e}_H, e_H)$  (This is off the equilibrium path).
  - A type  $\theta_L$  can guarantee a payoff of  $\theta_L$  by following equilibrium strategies. The deviation can bring type  $\theta_L$  at most:

$$\theta_H - c(e'|\theta_L) < \theta_L$$

- Thus, a type  $\theta_L$  would never deviating to  $e'$ . Formally  $e'$  is dominated in equilibrium for type  $\theta_L$ .
- The PBE does not pass the ICT: If  $\mu(e') = 1$ , type  $\theta_H$  would benefit from deviating to  $e'$ .

## Intuitive criterion: separating equilibrium



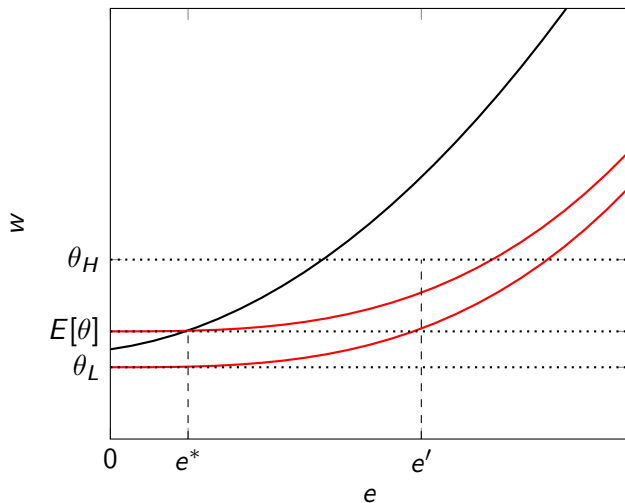
## Intuitive criterion: pooling equilibrium

- Let start instead from a pooling equilibrium at  $e^*$ .
- **Claim:** there exists  $e'$  such that

$$E[\theta] - c(e^*|\theta_H) < \theta_H - c(e'|\theta_L) < E[\theta] - c(e^*|\theta_L)$$

- Deviating to  $e'$  is dominated in equilibrium for type  $\theta_L$ .
- Thus, the pooling PBE does not pass the ICT.
  - If  $\mu(e') = 1$ , type  $\theta_H$  would benefit from deviating to  $e'$ .

## Intuitive criterion: pooling equilibrium



## Intuitive criterion

- Only the best separating PBE passes the ICT.
- Notice that sometimes *forced pooling* generates a Pareto improvement.
  - In particular, when the share of high types is sufficiently large.
- Another Pareto improvement can arise with *cross-subsidization*.

## Model with continuum of types

- Consider a model with a continuous of types.
  - Support in  $[\underline{\theta}, \bar{\theta}]$ .
  - Density function  $f$  str. positive everywhere in the support.
  
- **Question:** Is there a separating equilibrium? Is it unique?
  - Parametric assumption:  $c(e|\theta) = \alpha \cdot e^2/\theta$ .

# Empirical evidence

- Bedard (2001) “Human Capital Versus Signaling Models”
  - Study education as a signal of ability, exploiting the effect of constraining access to university in high school graduation levels.
  - **Empirical finding:** Regions with universities have higher high-school drop-out rates.
    - Difficult to explain in a model of human capital.
  - **Signaling explanation:**
    - With no university nearby, more high-ability students stop their education after completing high-school.
    - Low-ability students have incentives to finish high-school to pool with high-ability students.
  - **Policy implications:**
    - Improving access to university might increase drop-out rates and depress wages for some kids.

## Other models related to signaling

- Evidence and voluntary disclosure of verifiable information. Grossman (1981) Milgrom (1981) Dye (1985)
- Costless signaling (cheap talk): might work if preferences between sender and receiver are partially aligned. (Crawford Sobel (1982))



# Classical evidence models

Seminal model developed by Grossman (1981) and Milgrom (1981)

- Similar to the previous model.
  - One worker, more than 2 firms.
  - Worker has private type  $\theta$  with cdf  $F$ .
  - Firms compete offering wages.
  
- Instead of choosing a level of education, worker can take a (free) test that perfectly reveals his type.
  - Formally, worker can send a message in  $\{\emptyset, \theta\}$ .
  - Firms observe the message before making wage offers.

# Unraveling

- Let  $w(m)$  be the wage that firms offer to an agent that sends message  $m$ .
- Let  $\Theta_o$  be the subset of types that chooses the empty message in equilibrium.
- **Claim:** almost all types take the test:  $\Theta_o \subseteq \{\underline{\theta}\}$ 
  - Suppose that  $w(\emptyset) > \underline{\theta}$ .
  - It must be that  $\Theta_o = [\underline{\theta}, w(\emptyset))$
  - $w(\emptyset) = E[\theta|\Theta_o] < w(\emptyset)$ . Abs!
  - So  $w(\emptyset) = \underline{\theta}$  and  $\Theta_o \subseteq \{\underline{\theta}\}$ .

# Partial unraveling

- Dye (1985) and Jung and Kwon (1988): Worker has evidence with some probability  $\lambda$ , and no evidence otherwise (independent of type).
- **Partial unraveling:**
  - Let  $w$  be the wage for a worker in the absence of evidence.
  - Any type with  $\theta < w$  will not present evidence.
  - Equilibrium  $w$  is the unique solution to:

$$\begin{aligned}w &= E[\theta | m = \emptyset] &= E[\theta | \text{no evidence or } \theta < w]. \\ & &= q(w) \cdot E[\theta] + (1 - q(w)) \cdot E[\theta | \theta < w]\end{aligned}$$

where  $q(w) = Pr(\text{no evidence} \mid \text{no evidence or } \theta < w)$ .

## Partial unraveling

- Example:  $\theta \sim U[0, b]$ .

$$q(w) = \frac{p}{p + (1-p)F(w)} = \frac{p \cdot b}{p \cdot b + (1-p) \cdot w}$$

- So,

$$E[\theta | m = \emptyset] = \frac{p \cdot b}{p \cdot b + (1-p) \cdot w} \cdot \frac{b}{2} + \frac{(1-p)w}{p \cdot b + (1-p) \cdot w} \cdot \frac{w}{2}$$

- Solving  $E[\theta | m = \emptyset] = w$  we get

$$w = \frac{\sqrt{p} \cdot b}{1 + \sqrt{p}}$$