Advanced Microeconomics III

Screening

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Competitive screening

- **Spence's signaling model**: informed players (workers) move first.
- **Competitive screening**: uninformed player moves first.
  - This assumption seems more appropriate in some applications.

- **Example - insurance contracts.**
  - Insurance companies do not observe customers' risk types.
  - Companies offer various contracts to induce self-selection.
  - Different types accept different contracts.

- Other examples: labor market, lending markets.

Model

- Environment similar to Spence's model.

- **Agents**:
  - One worker and two firms.

- **Worker Types**:
  - $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L$.
  - Only the worker knows θ.
  - Firms assign probability q to type θ_H.

Contracts and payoffs

A *contract* is a pair $(e, w) \in \mathbb{R}^2_+$ that stipulates a wage w and an effort level e.

- **Payoffs under contract (w,e)**

$$\begin{aligned} \text{Firm that hires the worker:} &\qquad \theta - w \\ \text{Worker that is hired:} &\qquad w - c(e|\theta) \end{aligned}$$

Where $c(e|\theta)$ satisfies:

- $c(0|\theta) = 0$ for θ.
- $c(\cdot|\theta)$ str. increasing and str. convex.
- Single-crossing condition.

Timing

- **Timing**:
  1. Firms make simultaneous contract offers.
     Each firm may offer as may contracts as it wishes.
  2. Nature chooses the worker's type.
  3. Worker accepts one contract or rejects all of them.

(Pure-strategy) subgame-perfect Nash equilibrium

- A SPNE is described by:
  - The set of contracts offered by each firm, C_1 and C_2.
  - The acceptance decisions of the two worker types.

- Let $C = C_1 \cup C_2 \cup (0,0)$ be the set of available contracts.

- **Equilibrium conditions**:
  - Worker chooses (in any subgame) a contract that from

$$\arg\max_{(e,w)\in C} w - c(e,\theta)$$

  - No firm can increase its expected utility by offering a different set of contracts.

Monotonicity

Lemma

Consider any pure-strategy NE of any subgame where C is the set of available contracts. And let (e_L, w_L) and (e_H, w_H) denote contracts chosen by the two worker types. Then $e_H \geq e_L$.

Proof.

- Both contracts are optimal:

$$w_H - c(e_H|\theta_H) \geq w_L - c(e_L|\theta_H) \qquad \text{(IC-H)}$$

$$w_L - c(e_L|\theta_L) \geq w_H - c(e_H|\theta_L) \qquad \text{(IC-L)}$$

- Rearranging:

$$c(e_H|\theta_H) - c(e_L|\theta_H) \leq w_H - w_L \leq c(e_H|\theta_L) - c(e_L|\theta_L)$$

Monotonicity

Proof (Cont.)

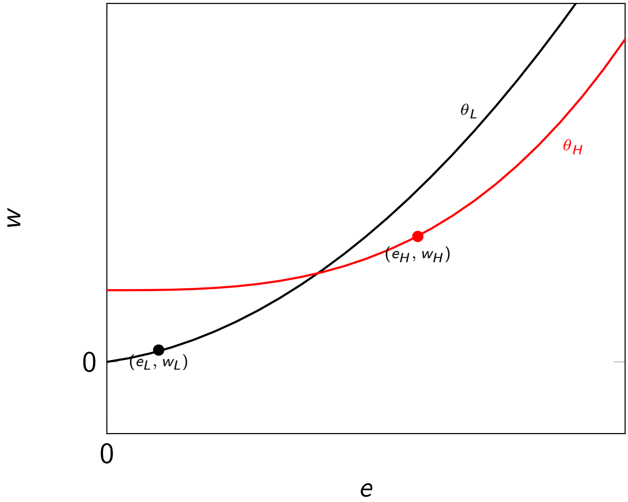
- Suppose that $e_H < e_L$. Then

$$
\begin{aligned}
c(e_L|\theta_H) - c(e_H|\theta_H) &= \int_{e_H}^{e_L} c'(e|\theta_H)\, de \\
&< \int_{e_H}^{e_L} c'(e|\theta_L)\, de \\
&= c(e_L|\theta_L) - c(e_H|\theta_L)
\end{aligned}
$$

- which contradicts the IC contstraints from before.

□

Monotonicity



Zero profits

Lemma

In any SPNE, both firms earn zero profits.

Proof.

- Suppose that firms' aggregate profit $\Pi > 0$.
- At least one firm's profit must be $\leq \Pi/2$, say firm 1's.
- Let (e_L, w_L) and (e_H, w_H) denote the respective contracts chosen by the two worker types.

Zero profits

Proof (Cont.)

- **Case 1**: $(e_L, w_L) = (e_H, w_H)$.
  - Then 1 can deviate to $C_1' = \{(e_L, w_L' + \epsilon)\}$ for small $\epsilon > 0$.
  - Firm 1's resulting profit is Π because it attracts both types.
  - This deviation is profitable.
- **Case 2**: $(e_L, w_L) \neq (e_H, w_H)$.
  - Firm 1 can deviate to $C_1' = \{(e_L, w_L + \epsilon_L), (e_H, w_H + \epsilon_H)\}$
  - Firm 1 can choose ϵ_L and ϵ_H so that the incentive constraints are satisfied with strict inequalities.
  - Firm 1's resulting profit is Π because it attracts both types.
  - This deviation is profitable.

□

No pooling equilibria

- **Pooling equilibrium**: SPNE in which both worker types choose a contract with the same effort level.

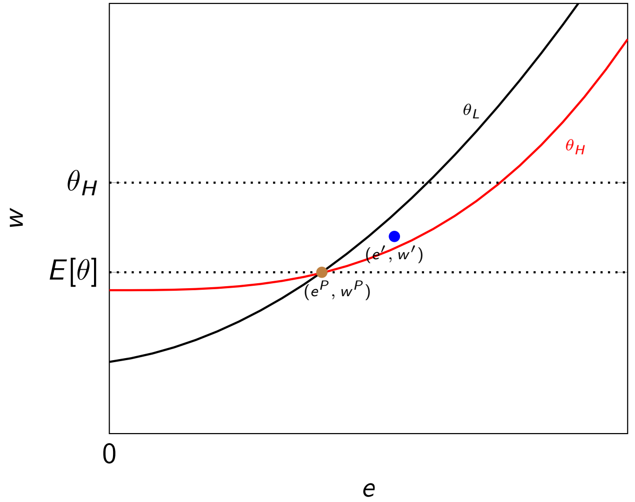
Proposition

There is no pooling equilibrium.

Proof.

- Suppose that there exists a SPNE in which both workers choose (e^P, w^P).
- Zero profit condition: $w^P = E[\theta] < \theta_H$.
- There exists a contract (e', w') that attracts only the H worker and such that $w' < \theta_H$.

No pooling equilibria



No Pareto efficient equilibrium

Corollary

There is no Pareto efficient SPNE.

Proof.

- Observe that an allocation is Pareto efficient if and only if both types choose contracts with zero effort.
- But we show that there is no pooling equilibrium.

□

Each chosen contract yields zero profit

Lemma

If (e_L, w_L) and (e_H, w_H) are the contracts chosen by the L and H-type workers in a SPNE, then $w_L = \theta_L$ and $w_H = \theta_H$.

Proof.

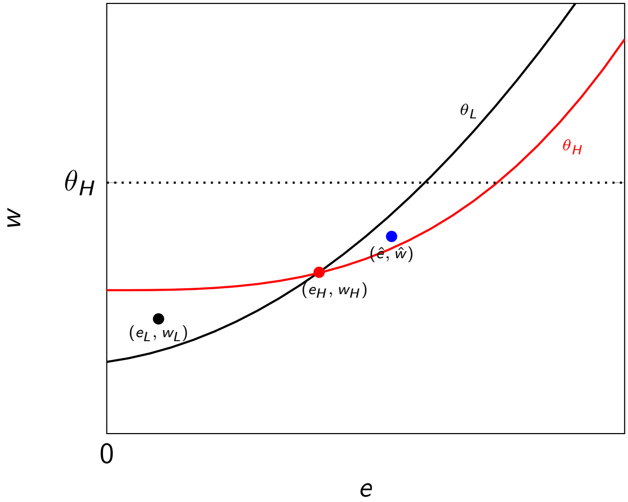
- $w_L \geq \theta_L$.
  - Proof by contradiction. Suppose $w_L < \theta_L$.
  - Consider a firm deviates to $C' = \{(e_L, w_L + \epsilon)\}$.
  - Then all L workers (and possibly the H workers) choose this contract.
  - For low epsilon, the deviation yields a positive profit because $w_L + \epsilon < \theta_L$.
  - But in equilibrium firms' profits must be zero, so this is a contradiction.

Each chosen contract yields zero profit

Proof (Cont.)

- $w_H \geq \theta_H$.
  - By contradiction: if $w_H < \theta_H$ then one firm has a profitable deviation to $(\hat{e}, \hat{w})$ with
    - $\hat{e} > e_H$.
    - $\hat{w} \in (w_H, \theta_H)$ such that this is attractive for the high type but not for the low type.

Each chosen contract yields zero profit



Each chosen contract yields zero profit

Proof (Cont.)

- We showed that $w_L \geq \theta_L$ and $w_H \geq \theta_H$.
- Finally, it must be that $w_L = \theta_L$ and $w_H = \theta_H$ because otherwise at least one firm would incur a loss.

□

L-worker's contract

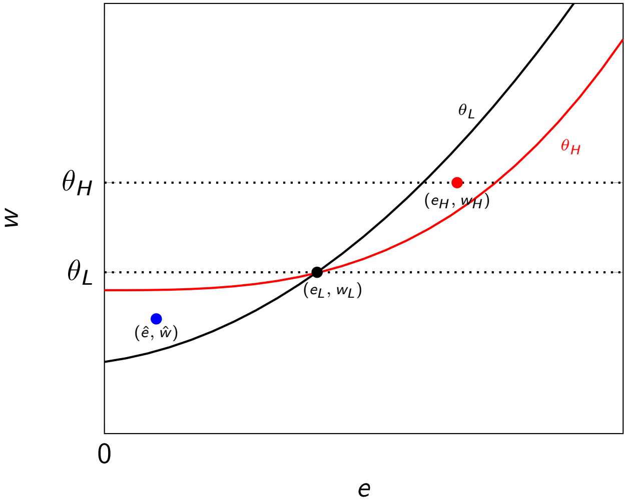
Lemma

In any SPNE, the L-worker accepts the contract $(0, \theta_L)$.

Proof.

- From previous result, in any SPNE the L worker chooses a contract (e_L, θ_L) for some $e_L \geq 0$.
- Because $\theta_L \neq 0$, this is not the outside option, i.e. it is offered by at least one firm, say, firm 1.
- Suppose that $e_L > 0$. Then firm 2 has a profitable deviation $C' = \{\hat{e}, \hat{w}\}$. See next figure.

L worker's contract



H-worker's contract

Lemma

In any SPNE, the H-worker accepts the contract (e_H^*, θ_H)*, where* e_H^* *satisfies*

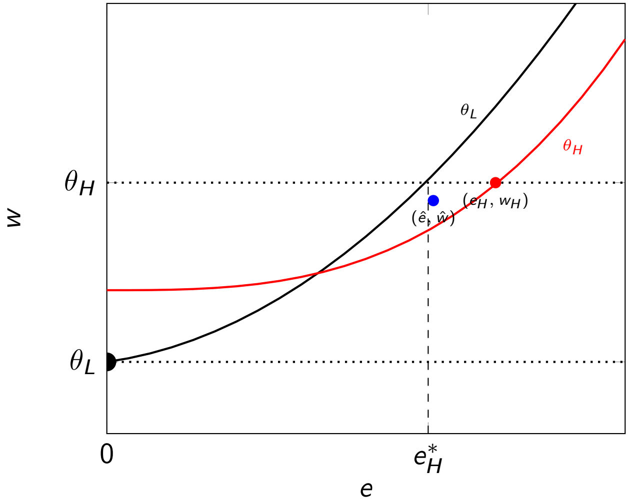
$$\theta_H - c(e_H^*, \theta_L) = \theta_L - c(0, \theta_L).$$

Proof.

- by IC of L-type, it must be that $e_H \geq e_H^*$.
- Suppose H worker accepts a contract (e_H, θ_H) with $e_H > e_H^*$.
- At least one firm i anticipates that the other firm offers $(0, \theta_L)$.
- Firm i has a profitable deviation. See next figure.

□

H-worker's contract



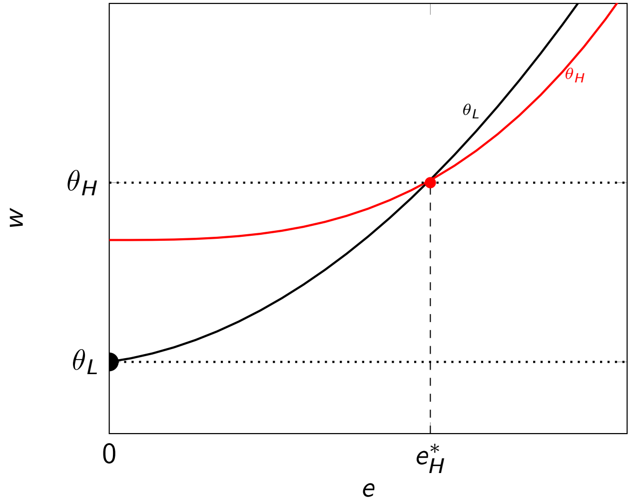
Unique equilibrium candidate

We can summarize all previous results as follows:

Proposition

If there exists a SPNE, then it yields the same outcome as the best separating equilibrium in the Spence model.

Unique equilibrium candidate



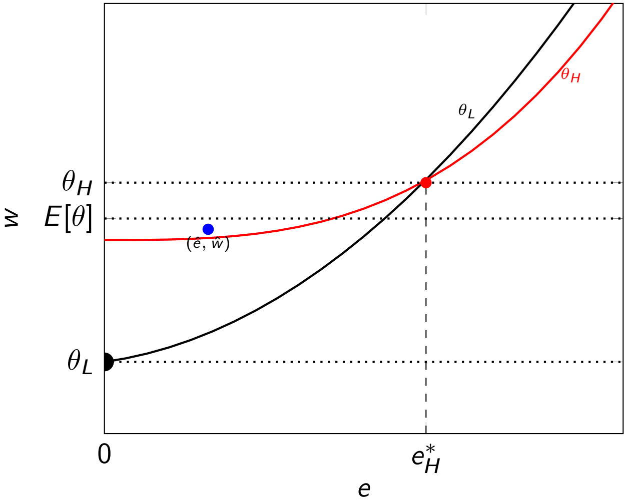
Equilibrium existance

Proposition

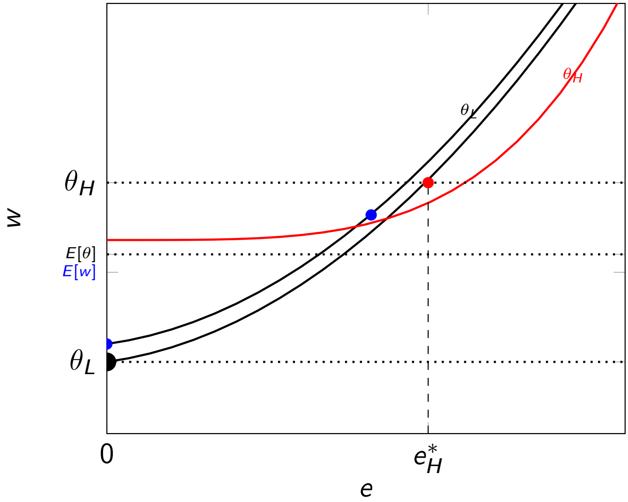
A SPNE exists if and only if the fraction q of H-workers is sufficiently small.

- In the candidate equilibrium, there is no single-contract deviation that attracts only one type of worker and is profitable.
- But there can be a single-contract deviation that attracts both types and is profitable ("pooling deviation").
- Also, there can be a two-contract deviation such that each contract attracts one type ("cross-subsidizing deviation").
- None of these deviations is profitable if and only if q is sufficiently small.

Pooling deviations



Cross-subsidizing deviations



Constrained Pareto optimality

An ordered pair of contracts $((e_L, w_L), (e_H, w_H))$ is *incentive compatible* (IC) if each type prefers the corresponding contract.

A IC pair of contracts is C *weakly constrained Pareto optimal* if there is no IC pair of contracts C' that both workers types an the firms (in aggregate) are strictly better off if C' is iffered instead of C.

Proposition

If a SPNE exists, then the corresponding equilibrium contracts are weakly contrained Pareto optimal.

Constrained Pareto optimality

Proof.

- Assume that SPNE exists, suppose that there exists an IC pair C' such that everybody is strictly better off. Then either:
  - C' is a singleton, and thus a profitable deviation for each firm.
  - Or a perturbation of C' such IC are satisfied strictly is a profitable deviation for each firm.

□

Wilson equilibrium

A set of contracts is a Wilson Equilibrium if there is no profitable deviation that remains profitable once unprofitable offers have been withdrawn.

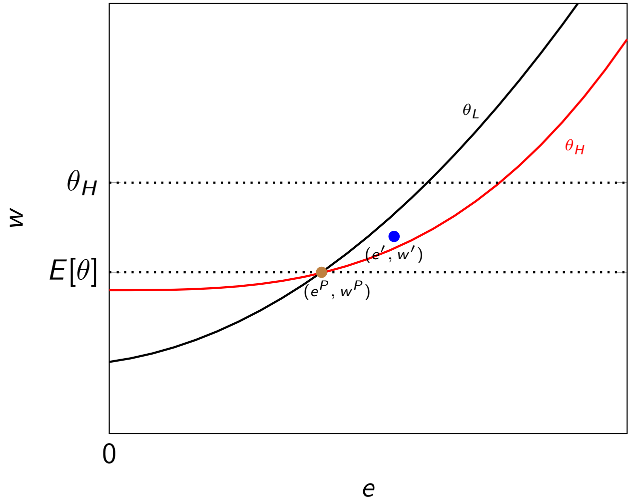
Theorem

If the share of H-types is high enough, there exists a Pareto efficient Wilson equilibrium.

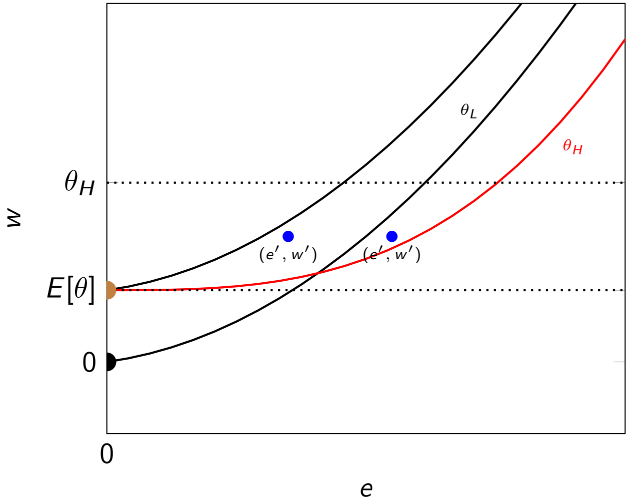
Pareto efficient Wilson equilibrium

- Before, we rule out all pooling equilibria with a deviation that attracts only the high type.
- This deviation is not "profitable" in the Wilson sense.

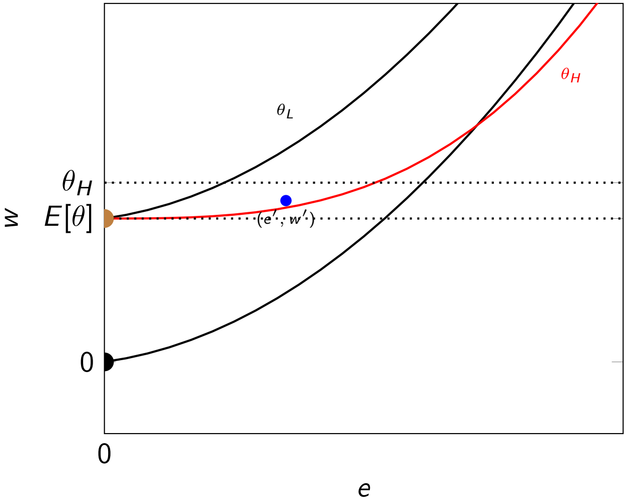
Pareto efficient Wilson equilibrium



Pareto efficient Wilson equilibrium



Pareto efficient Wilson equilibrium



Mixed-strategy equilibria

- Some properties that we derived for the equilibrium continue to hold when we consider mixing.
  - Zero profits (ex-ante)
  - $e_L = 0$.

Proposition (Rosenthal and Weiss (1984))

A symmetric mixed strategy equilibrium exists. In it:

- Both firms mix over a set of separating contracts that yield zero profit in expectation to the firm.
- Each contract in the support has $e_L = 0$.