Problem Set 1

Advanced Microeconomics III

FSS 2023

Problem 1 Based on MWG 13.B.3.

Consider a positive selection version of the adverse selection model in which $r(\cdot)$ is continuous and strictly decreasing. Also assume that F has a strictly positive density on $[\underline{\theta}, \bar{\theta}]$

a. Show that the *more capable* workers are the ones choosing to work for any given wage.

b. Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

c. Show that in any competitive equilibrium the trading activity is inefficiently high if the following assumption is satisfied:

- there exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $r(\theta) < \theta$ for all $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for all $\theta < \hat{\theta}$.

Problem 2 Based on MWG 13.B.6 and Wilson (1980).

Consider the following extension of the adverse selection model. There is a mass N' of buyers, each of which wants to buy at most one car. The buyers differ in their willingness to pay for the car: a buyer of type γ has the willingness to pay $\gamma\theta$ for a car bought from a seller of type θ. Be aware that now each seller has some private information θ and each buyer has some private information γ. Assume that γ is distributed with a strictly positive density function g on $[0, \infty)$; let G denote the corresponding cumulative distribution function.

a. Let $z(p, \mu)$ denote the aggregate demand for cars when the price is p and the average quality of cars offered at price p is μ. Derive an expression for the function z in terms of G.

b. Let $\mu(p) = E[\theta | r(\theta) \leq p]$ and define the aggregate demand for cars by $z^*(p) = z(p, \mu(p))$. Assuming that μ is differentiable, show that z^* is strictly increasing around a point $\bar{p}$ if, at $p = \bar{p}$, the elasticity of μ with respect to p exceeds 1, and is strictly decreasing if the elasticity is below 1. Interpret!

c. Assume that r is strictly increasing and continuous. Let $s(p) = N \cdot F(r^{-1}(p))$ denote the aggregate supply of cars, and define a competitive equilibrium price p^* by the equation $z^*(p^*) = s(p^*)$. Show that if there are multiple competitive equilibria, then the one with the highest price Pareto dominates all others.

d. Consider a game-theoretic model in which buyers make simultaneous price offers. Show that (1) only the highest competitive equilibrium price can arise as a SPNE and (2) the highest-price competitive equilibrium p^* is a SPNE if and only if $z^*(p) \leq z^*(p^*)$ for all $p > p^*$.

Problem 3 Consider the Adverse Selection model with the following modification: The population distribution of qualities is uniform in $[0, 1]$. There are two types of sellers L and H. The seller's type is independent of his quality θ and the proportion of H-type sellers in the population is $\beta \in (0, 1)$.

a. Assume that the reservation value of a seller of type L is $r_L(\theta) = \alpha_L \cdot \theta$ and the reservation value of a seller of type H is $r_H(\theta) = \alpha_H \cdot \theta$ with $\alpha_L < \alpha_H < 1$. For which parameters α_H, α_L, and β is there a competitive equilibrium with a positive mass of trade?

b. Assume that the reservation value of a seller of type L is $r_L(\theta) = \alpha_L \cdot \theta$ with $\alpha_L < 1$ and the reservation value of a seller of type H is $r_H(\theta) = \bar{r} < 1$. For which parameters $\bar{r}, \alpha_L$, and β is there a competitive equilibrium with a positive mass of trade?

Problem 4 (Optional)

Consider the adverse selection model and assume that the distribution of θ is exponential with parameter λ.

a. Write down an expression for $E[\theta | \theta < \hat{\theta}]$.

b. Assume that $r(\theta) = \alpha\theta$.

i. For which α there exists a CE involving a complete market breakdown?

ii. For which α there exists a CE without a complete market breakdown?