Problem Set 3

Spring 2023

Advanced Microeconomics III

Problem 1 (Exam 2022)

Consider a variation of the signaling model covered in class: There is a worker of type $\theta \in \{\theta_L, \theta_H\}$, where $\theta_H > \theta_L$. The worker knows their type, but firms do not. If employed by a firm, the worker produces output θ. The worker moves first, choosing a publicly observable level of education $e \in [0, \infty)$. The cost of education $c(e)$ is the same for both types (increasing, with $c(0) = 0$).

After the worker chooses their level of education—but before firms compete to hire the worker—a publicly observable signal perfectly reveals the type of the worker with probability $p(e)$. With probability $(1 - p(e))$ the signal is empty. Assume that $p(\cdot)$ is continuous, differentiable, and strictly increasing with $p(0) = 0$.

The payoff of a worker that obtains a wage w and chooses education e is

$$u(w, e) = w - c(e)$$

after observing the education chosen by the worker and the public signal (if available), firms simultaneously offer wages. The payoff of the firm that hires a worker of type θ and pays a wage w is $\theta - w$.

The solution concept is PBE with symmetric beliefs among firms. Additionally, we impose that when firms observe the type of worker through the public signal, beliefs are consistent with the signal (i.e. the market disregards any inferences derived from the worker's education choice).

a. Let w_H and w_L be the respective equilibrium wages for a worker of type θ_L and θ_H after the public signal reveals the worker's type (given the assumption on beliefs out of the equilibrium path, these wages do not depend on e). Write w_H and w_L as a function of the primitives of the model.

b. In equilibrium, the wage a worker receives when the type is not revealed by the signal depends on the inferences that firms make given the level of education chosen by the worker. For the next two subproblems, consider the wage that a worker receives when the types are not revealed to be fixed at w°.

i. What is the expected payoff of a worker that chooses a level of education e as a function of w° and the primitives of the model?

ii. Show that the preferences of workers over (e, w°) satisfy that the indifference curves are flatter for type θ_H (with w° in the vertical axis and e in the horizontal axis).

c. Find the smallest education level for type θ_H that supports a separating pure-strategy equilibrium.

d. Under what conditions on the model's primitives does a pooling equilibrium with both types selecting zero education exist? Give a more precise condition for the parametric case in which $p(e) = \frac{e}{1+e}$ and $c(e) = k \cdot e$.

Problem 2 Based on MWG 13.D.2.

Consider the following model of an insurance market. There are two types of individuals: high-risk and low-risk. Each individual starts with an initial wealth W. An accident (e.g., fire) reduces the individual's wealth by K. The probability of the accident happening is p_L for low-risk types and p_H for high-risk types, where $p_H > p_L$. Each individual is privately informed about her risk type $p \in \{p_L, p_H\}$.

Individuals are expected-utility maximizers with a Bernoulli utility function $u(w)$ over wealth w. Assume that $u'(w) > 0$ and $u''(w) < 0$ for all w. There are two risk-neutral insurance companies that can offer one or more insurance policies ("contracts"). A contract consists of a premium payment M made by the insured individual to her insurance firm, and a payment R from the insurance company to the insured individual in the event of a loss. Consider a game where the insurance companies first simultaneously offer sets of contracts. Second, nature independently draws a type for each individual, where the probability of low-risk type is $\lambda \in (0, 1)$. Third, each individual accepts at most one contract.

a. What is the expected utility of an individual of type p who accepts a contract $c = (R, M)$? What is the expected payoff for a firm selling the contract c to an individual of type p?

b. In an (R,M) diagram with R on the horizontal axis, sketch a firm's zero-profit line when contracting with a high-risk type, and its zero-profit line when contracting with a low-risk type. Sketch an indifference curve for each type.

Show that for any contract c, the high-risk indifference curve through c is steeper than the low-risk indifference curve.

c. Describe the unconstrained Pareto efficient allocations.

d. Describe the unique pure-strategy subgame-perfect Nash equilibrium candidate.

e. Show by example that it is possible that no equilibrium exists.